Math 250 Optimization or Applied Maxima/Minima Questions (4.4)

Process for Solving:

Step 0: Read the problem carefully to identify the variables and organize the give information. Draw a picture.

Step 1: Identify the word describing the primary **objective** function to be optimized (maximized or minimized) using words like most, least, greatest, largest, and smallest. Write this primary function. [Note: The problem will NOT give you a value for this function, and at this step, your function will have more than two variables.]

Step 2: Identify the word describing the secondary **constraint** equation(s) and its value, to be used to eliminate a variable from the primary function. Write the equation and set it equal to the given value.

[Caution: It is impossible to get the correct answer if you mix up the primary and secondary functions.]

Step 3: Determine the independent variable needed in the final answer. Solve the secondary equation for the other variable, in terms of the desired independent variable.
Step 4: Substitute the result from solving the secondary equation into the primary function, and simplify.
Step 5: Differentiate the primary function, find its critical values, and test the CVs using the first or second derivatives test to find the desired maximum or minimum.
Step 6: Write the answers to all questions asked in the problem, including units.

Examples and Practice:

1) A closed rectangular container with square base is to have a volume of 2250 cubic inches. The material for the top and bottom of the container will cost \$2 per square inch, and the material for the sides will cost \$3 per square inch. Find the dimensions of container of least cost.
2) A rectangle is bounded by the x and y-axes and the graph of $y = \frac{6-x}{2}$. What length and width should the rectangle have so its area is a maximum?
3) Determine the dimensions of a rectangular solid (with square base) with maximum volume if its surface area is 337.5 square centimeters.
4) A rancher has 400 feet of fencing with which to enclose two identically-sized adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?
5) A rectangular plot of land is to be fenced using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 per foot, while the remaining two sides will be fenced with standard fencing selling for \$2 per foot. What are the dimensions of maximum area that can be fenced at a cost of exactly \$6000?
6) Find the point on the graph of $f(x) = \sqrt{x-8}$ which is closest to the point (12,0).
7) An industrial tank is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 4000 cubic feet. Find the radius of the cylinder that has minimum surface area.
8) A rectangular page is to contain 36 square inches of print. The margins on each side are $1\frac{1}{2}$ inches. Find the dimensions of the page such that the least amount of paper is used.
9) Find the speed v, in miles per hour, that will minimize costs on a 110-mile delivery trip. The cost per hour for fuel is $C(v) = \frac{v^2}{500}$ and the driver is paid \$7.50 per hour. Assume there are no other costs.

Units Analysis for Word Problems

The following word problems make very little sense. But they do have enough information to determine some critical parts of the question. For each "problem", write the concept being calculated, the formula, and an equation.

Example:

1) A rectangular whiziwig is to be schmootzled with 300 square feet of biddledunk.

Rectangular Area = LW = 300 square feet

2) For a total cost of $300, a rectangular difgik is gikdiffed at $2/ft for the long sides, but $3/ft on the short sides.

Length times cost per unit of length

$$\text{Cost} = \$300 = \frac{\$2}{foot}\cdot 2L\ feet + \frac{\$3}{foot}\cdot 2W\ feet \leftarrow \text{units of feet cancel out}$$

Cost =4L+6W=$300

Practice:

3) Fluntsing a circular thingamabob requires 300 square meters of squelzer.
4) Total cost $200, cube $2/cubic foot
5) Total cost $300, rectangle, $2/sq ft
6) While yalping a circular cosita, the tringle used 300 miles of buffling.
7) Total cost $300, sphere, $2/cubic foot
8) Total cost $300, square, $2/ft
9) A spherical howsit contains 300 cubic millimeters of gwandle.
10) Total cost $300, rectangular solid, $2/cubic foot
11) Total cost $300, circular, $2/ft
12) A square doodad is to be grinzled with 300 square centimeters.
13) Total cost $300, cube $2/sq foot
14) Ronselizing a rectangular whatchamacallit takes 300 yards of hangle.
15) Total cost $300, sphere, $2/sq foot
16) A rectangular whatsit, while not in phitrit, holds 300 cubic decimeters.
17) Total cost $300, rectangular solid, $2/sq foot
18) Square whosits in a luquerizer need 300 kilometers of chitterizer.
19) Total cost $300, square, $2/sq ft
20) While yoodling a cubic no-sé-qué, 300 cubic inches of jooblers are bekrumpled.
21) Total cost $300, circle, $2/sq ft

Units Analysis for Word Problems

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2) For a total cost of \$300, a rectangular difgik is gikdiffed at \$2/ft for the long sides, but \$3/ft on the short sides.

Length times cost per unit of length → based on perimeter, but not exactly the same.

$$\text{Cost} = \$300 = \frac{\$2}{\cancel{\text{foot}}}\cdot 2L\ \cancel{\text{feet}} + \frac{\$3}{\cancel{\text{foot}}}\cdot 2W\ \cancel{\text{feet}}$$ ← units of feet cancel out $\frac{ft}{ft}=1$

Cost =4L+6W=\$300

Practice:

3) Fluntsing a circular thingamabob requires 300 square meters of squelzer. $\boxed{\pi r^2 = 300}$
4) Total cost \$200, cube \$2/cubic foot volume $V=s^3$ ⇒ cost $\frac{\$2}{ft^3}\cdot s^3 ft^3$ ⇒ $\boxed{2s^3 = 200}$
5) Total cost \$300, rectangle, \$2/sq ft area $A=L\cdot W$ ⇒ cost $\frac{\$2}{ft^2}\cdot LW\,ft^2$ ⇒ $\boxed{2LW = 300}$
6) While yalping a circular cosita, the tringle used 300 miles of buffling. circumference $C=\boxed{2\pi r = 300}$
7) Total cost \$300, sphere, \$2/cubic foot Volume $V=\frac{4}{3}\pi r^3$ ⇒ cost $\frac{\$2}{ft^3}\cdot \frac{4}{3}\pi r^3 ft^3$ ⇒ $\boxed{\frac{8}{3}\pi r^3 = 300}$
8) Total cost \$300, square, \$2/ft Perimeter $P=4s$ ⇒ cost $\frac{\$2}{ft}\cdot 4s\,ft$ ⇒ $\boxed{8s = 300}$
9) A spherical howsit contains 300 cubic millimeters of gwandle. Volume $=\boxed{\frac{4}{3}\pi r^3 = 300}$
10) Total cost \$300, rectangular solid, \$2/cubic foot Volume $= L\cdot W\cdot H$ ⇒ cost $=\frac{\$2}{ft^3}\cdot LWH ft^3 = \boxed{2LWH = 300}$
11) Total cost \$300, circular, \$2/ft Circumference $C=2\pi r$ ⇒ Cost $\frac{\$2}{ft}\cdot 2\pi r\,ft = \boxed{4\pi r = 300}$
12) A square doodad is to be grinzled with 300 square centimeters. Surface area $A=\boxed{6s^2 = 300}$
13) Total cost \$300, cube \$2/sq foot surface area $S=6s^2$ ⇒ cost $\frac{\$2}{ft^2}\cdot 6s^2 = \boxed{12s^2 = 300}$
14) Ronselizing a rectangular whatchamacallit takes 300 yards of hangle. $P=\boxed{2L+2W = 300}$
15) Total cost \$300, sphere, \$2/sq foot (Surface area $4\pi r^2$ ⇒ cost $2\cdot 4\pi r^2 = \boxed{8\pi r^2 = 300}$
16) A rectangular whatsit, while not in phitrit, holds 300 cubic decimeters. $\boxed{LWH = 300}$
17) Total cost \$300, rectangular solid, \$2/sq foot $A=L\cdot W$ ⇒ cost $\boxed{2LW = 300}$
18) Square whosits in a luquerizer need 300 kilometers of chitterizer. $P=\boxed{4s = 300}$
19) Total cost \$300, square, \$2/sq ft $A=s^2$ ⇒ cost $\boxed{2s^2 = 300}$
20) While yoodling a cubic no-sé-qué, 300 cubic inches of jooblers are bekrumpled. $V=\boxed{s^3 = 300}$
21) Total cost \$300, circle, \$2/sq ft $A=\pi r^2$ ⇒ cost $=\boxed{2\pi r^2 = 300}$

Math 250 Optimization Problems

Objective: 1) Solve applied maximum and minimum problems.

*Maximum or Minimum = Optimum, or best.
To find either the maximum or minimum is to "optimize", or find the optimum.

Step 1: Identify the concept we wish to optimize (find max or min), and write an equation.
This is called the primary function or equation because it's the most important.
We will use calculus on this function to find its max or min.

CAUTION: The primary equation will almost always have too many variables. (typically 3, total)

Step 2: Identify another function or equation in the given information and write it down.
This is called the secondary function or equation because it's used only to simplify the primary one.
We will use algebra on this function — but no calculus.

Step 3: Identify the quantity and its variable which are needed to answer the question. We'll call this the desirable variable, because we want it to remain in the work until the end.

Step 4: Identify the quantity and its variable which are not the concept to be optimized and not the desirable variable. This extra variable we'll call the undesirable variable, because we want to get rid of it before we optimize (do calculus).

Step 5: Solve the secondary equation for the undesirable variable and simplify

Step 6: Substitute the result from Step 5 into the primary equation. and simplify.

CHECK: After Step 6, the primary equation should have only two variables: 1. the variable to be optimized 2. the desirable variable (for the answer)

Now optimize:

Step 7: Find the first derivative of the primary function using result from Step 6.

> CAUTION: Take the derivative of the concept to be optimized with respect to the desirable variable. This is rarely a simple x and y.

Step 8: Find the critical values by setting the first derivative from Step 7 equal to zero and solving the resulting equation.

Step 9: Check that each critical value (of the desirable variable) makes physical sense for the problem.
e.g. no negative, lengths.

Step 10: Use either the First Derivative Test or the Second Derivative Test (you'll have to find the second derivative of the primary function if so) to confirm whether the critical values from steps 8 & 9 are maxima or minima.

> CAUTION: You must do a test to get full credit!

Step 11: Check whether question asked for max or min and confirm which critical value is the answer.

Step 12: Answer the question — you may need to plug into the secondary or primary equations — and write your answer(s) with correct units.

Other hints

1) Brush up on memorizing all your geometry formulas.
2) Draw and label diagrams.
3) Make lists of variables and values, then label them as desirable or undesirable.

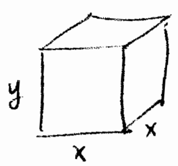
M250

1

A closed rectangular container with square base is to have a volume of 2250 cubic inches. The material for the top and bottom of the container will cost \$2 per square inch and the material for the sides will cost \$3 per square inche. Find the dimensions of container of least cost.

Primary: "least cost" $\Rightarrow$ minimize cost

Cost = (Cost of top) + (cost of bottom) + (cost of four sides)



$$= 2\cdot\underbrace{(x\cdot x)}_{\text{area top}} + 2\underbrace{(x\cdot x)}_{\text{area bottom}} + 3\underbrace{(4)}_{\text{4 sides}}\underbrace{(x\cdot y)}_{\text{area one side}}$$

$$= 2x^2 + 2x^2 + 12xy$$

$$= 4x^2 + 12xy$$ $\leftarrow$ Has two variables, x and y. Need to remove y.

Secondary: "volume of 2250 in^3"

$$V = x\cdot x\cdot y = 2250$$

$$x^2y = 2250$$ $\leftarrow$ solve for y

$$y = \frac{2250}{x^2}$$

Subst secondary into primary:

$$C(x) = 4x^2 + 12x\left(\frac{2250}{x^2}\right)$$

$$C(x) = 4x^2 + \frac{27000}{x} = 4x^2 + 27000x^{-1}$$

Find minimum:

$$C'(x) = 8x - 27000x^{-2} = 8x - \frac{27000}{x^2} = \frac{8x^3 - 27000}{x^2}$$

C.N's. $8x^3 - 27000 = 0$

$$8x^3 = 27000$$

$$x^3 = \frac{27000}{8}$$

$$x = \frac{30}{2} = 15$$

$x^2 = 0$

$x = 0$

Test:

c': $(-)$ | 0 | $(-)$ | 15 | $(+)$

$\searrow$ $\searrow$ $\nearrow$

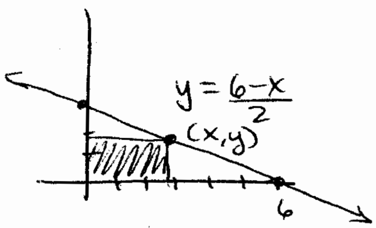
MIN

Subst back:

$$y = \frac{2250}{15^2} = 10$$

$\boxed{15 \text{ in} \times 15 \text{ in} \times 10 \text{ in}}$

(2) A rectangle is bounded by the x and y-axes and the graph of $y=(6-x)/2$. What length and width should the rectangle have so its area is a maximum?



$\text{Area} = x \cdot y$ ← Primary function too many variables

subst $y = \frac{6-x}{2}$ ← Secondary function used to remove y.

$\text{Area} = x \cdot \left(\frac{6-x}{2}\right)$

$A(x) = 3x - \frac{1}{2}x^2$ ← Primary function now has one variable.

optimize:

$A'(x) = 3-x$

$A'(x) = 0$ when $3-x=0$

$x=3$

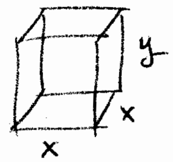
$\xleftarrow[\nearrow \quad 3 \quad \searrow]{+ \qquad\qquad -}$ $\quad A'(0) > 0$, $A'(4) < 0$ first derivative test.

relative max occurs when $x=3$.

$y = \frac{6-3}{2} = \frac{3}{2}$

length and width of rectangle: $\boxed{3 \times \frac{3}{2}}$

(3) Determine the dimensions of a rectangular solid (with square base) with maximum volume if its surface area is 337.5 cm².



volume $= x^2y$ ← Primary function. too many variables

surface area $= 2x^2 + 4xy = 337.5$ ← secondary function

solve for x or y, y looks easier:

$$4xy = 337.5 - 2x^2$$

$$y = \frac{337.5}{4x} - \frac{2x^2}{4x}$$

$$y = \frac{675}{8x} - \frac{x}{2}$$

$$y = \frac{675}{8}x^{-1} - \frac{x}{2}$$

subst into volume equation to remove y:

$$V = x^2\left(\frac{675}{8}x^{-1} - \frac{x}{2}\right)$$

$$V = \frac{675}{8}x - \frac{x^3}{2}$$ ← Primary function now has only one variable.

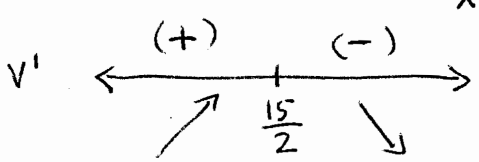
optimize:

$$V'(x) = \frac{675}{8} - \frac{3}{2}x^2 = 0$$

$$-\frac{3}{2}x^2 = -\frac{675}{8}$$

$$x^2 = \frac{225}{4}$$

$$x = \pm\frac{15}{2}$$ discard negative result. $\frac{15}{2} = 7.5$



$V'(1) > 0$

$V'(10) < 0$ first derivative test

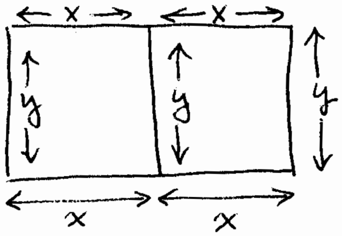
$x = \frac{15}{2}$ is location of max volume.

$$y = \frac{675}{8}x^{-1} - \frac{x}{2}$$ substitute back to find y.

$$y = \frac{675}{8}\cdot\frac{2}{15} - \frac{15}{4} = \frac{15}{2}$$

dimensions: $\boxed{\frac{15}{2}\text{cm} \times \frac{15}{2}\text{cm} \times \frac{15}{2}\text{cm}}$

(4) A rancher has 400 feet of fencing with which to enclose two adjacent rectangular corrals. What dimensions should be used so that the enclosed area will be a maximum?



Area = L · W

$A = 2x \cdot y$ ← Primary function too many variables.

Fencing $4x + 3y = 400$ ← Secondary function.

$3y = 400 - 4x$

$y = \dfrac{400 - 4x}{3}$.

Subst into A to remove the y-variable:

$A(x) = 2x \cdot \left(\dfrac{400 - 4x}{3}\right)$

$A(x) = \dfrac{800x}{3} - \dfrac{8x^2}{3}$ ← Primary function now has one variable

$A'(x) = \dfrac{800}{3} - \dfrac{16x}{3}$

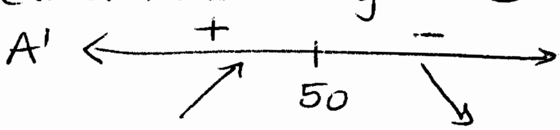
$A'(x) = 0$ when $\dfrac{800}{3} - \dfrac{16x}{3} = 0$

$\dfrac{16x}{3} = \dfrac{800}{3}$

$16x = 800$

$x = 50$

$A'(x)$ defined everywhere.



first derivative test

$A'(0) \geq 0$

$A'(100) < 0$

So $x = 50$ is a rel max.

OR $A''(x) = \dfrac{-16}{3} < 0$ everywhere

concave down

max.

second derivative test

Dimensions of pen x by y

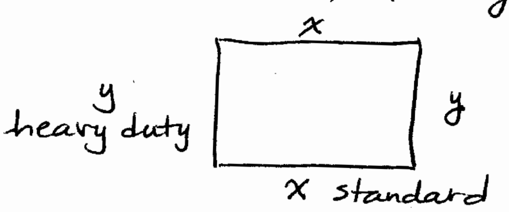
$y = \dfrac{400 - 4(50)}{3}$

$y = \dfrac{400 - 200}{3} = \dfrac{200}{3}$

Dimensions: of max area

$\boxed{x = 50 \text{ ft} \quad y = \tfrac{200}{3} \text{ ft or } 66\tfrac{2}{3} \text{ ft}}$

(5) A rectangular plot of land is to be fenced using two kinds of fencing. Two opposite sides will use heavy-duty fencing selling for \$3 per foot while the remaining two sides will be fenced with standard fencing selling for \$2 per foot. What are the dimensions of maximum area that can be fenced at a cost of exactly \$6000?



$\text{Area} = x \cdot y$ ← Primary function too many variables

$\text{Cost} = 2x \cdot 2 + 2y \cdot 3 = 6000$

$4x + 6y = 6000$ ← Secondary function.

Solve for y: $6y = -4x + 6000$

$y = -\frac{2}{3}x + 1000$

Subst into Area to remove y:

$A(x) = x\left(-\frac{2}{3}x + 1000\right)$

$A(x) = -\frac{2}{3}x^2 + 1000x$ ← Primary function now has only one variable.

Derivative to find C.V.'s and max/min:

$A'(x) = -\frac{4}{3}x + 1000$

$A'(x) = 0$

$-\frac{4}{3}x + 1000 = 0$

$1000 = \frac{4}{3}x$

$x = \frac{3000}{4} = 750$

Test:

A': (+) ↗ 750 ↘ (−)

MAX

or

$A''(x) = -\frac{4}{3} < 0$ for all x

concave down ⌒ MAX

Find other dimension by subst into Cost or Cost solved for y:

$y = -\frac{2}{3}(750) + 1000$

$y = 500$

Dimensions: $\boxed{\begin{array}{l}\text{750 ft standard fencing}\\ \text{500 ft heavy duty fencing}\end{array}}$

Examples

ⓑ p.223 #16.
Find the point on the graph of $f(x) = \sqrt{x-8}$ which is closest to the point $(12,0)$.

Distance formula $\quad D = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2} \leftarrow$ Primary function.

$(x_1, y_1) \to (12, 0)$

$(x_2, y_2) \to$ on graph $\to (x, \sqrt{x-8}) \leftarrow f(x) = \sqrt{x-8}$ is the Secondary function.
(x, y)

$$D(x) = \sqrt{(x-12)^2 + (\sqrt{x-8} - 0)^2}$$
$$D(x) = \sqrt{(x-12)^2 + x - 8}$$
$$D(x) = \sqrt{x^2 - 24x + 144 + x - 8}$$
$$D(x) = \sqrt{x^2 - 23x + 136}$$

Simplified Primary function has only one variable

Explicit derivative

$$D'(x) = \tfrac{1}{2}(x^2 - 23x + 136)^{-1/2}(2x-23)$$
$$D'(x) = \frac{2x-23}{2\sqrt{x^2-23x+136}}$$

Implicit derivative

$$D^2 = x^2 - 23x + 136$$
$$2D\frac{dD}{dx} = 2x - 23$$
$$\frac{dD}{dx} = \frac{2x-23}{2D}$$
$$\frac{dD}{dx} = \frac{2x-23}{2\sqrt{x^2-23x+136}}$$

critical values

$D'(x) = 0$ when $2x - 23 = 0$
$x = \frac{23}{2}$.

$D'(x)$ undef when $x^2 - 23x + 136 = 0$ (has no $\mathbb{R}$ zeros).

D': $\quad - \quad \frac{23}{2} \quad +$
$\quad \searrow \quad \frac{23}{2} \quad \nearrow$

$D'(0) \le 0$
$D'(12) > 0$

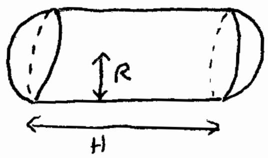
D'' ugly. ☹ use 1st derivative test!

$x = \frac{23}{2}$ is a relative min.

$$f\left(\tfrac{23}{2}\right) = \sqrt{\tfrac{23}{2} - 8}$$
$$= \sqrt{\frac{23-16}{2}}$$
$$= \sqrt{\frac{7}{2}}$$
$$= \frac{\sqrt{14}}{2}$$

$\boxed{\text{Point } \left(\frac{23}{2}, \frac{\sqrt{14}}{2}\right)}$ is closest to $(12, 0)$.

(7) An industrial tank is formed by adjoining two hemispheres to the ends of a right circular cylinder. The total volume of the solid is 4000 ft^3. Find the radius of the cylinder that has minimum surface area.



Minimize surface area:

$$A = \underbrace{4\pi r^2}_{\substack{\text{area of two hemispheres}\\ = \text{area of one sphere}}} + \underbrace{2\pi r h}_{\substack{\text{area of cylinder's}\\ \text{lateral side, no bases}}} \leftarrow \text{Primary function}$$

$A = 4\pi r^2 + 2\pi r h$ has 3 variables ☹

Volume: $4000 = \underbrace{\frac{4}{3}\pi r^3}_{\text{volume sphere}} + \underbrace{\pi r^2 h}_{\text{volume cylinder}} \leftarrow$ Secondary function.

Question says "find the radius" $\Rightarrow$ we want r, not h.
Solve for h in volume equation:

$$\frac{4000}{\pi r^2} - \frac{\frac{4}{3}\pi r^3}{\pi r^2} = \frac{\pi r^2 h}{\pi r^2}$$

$$\frac{4000}{\pi r^2} - \frac{4}{3}r = h$$

Subst into area function: (to remove h)

$$A(r) = 4\pi r^2 + 2\pi r\left(\frac{4000}{\pi r^2} - \frac{4}{3}r\right) \quad \text{dist}$$

$$= 4\pi r^2 + 8000 r^{-1} - \frac{8}{3}\pi r^2 \quad \text{combine like terms}$$

$$A(r) = \frac{4}{3}\pi r^2 + 8000 r^{-1} \leftarrow \text{Primary function has one variable.}$$

Optimize:

$$A'(r) = \frac{8}{3}\pi r - 8000 r^{-2}$$

$$A'(r) = 0$$

$$\frac{8}{3}\pi r = \frac{8000}{r^2}$$

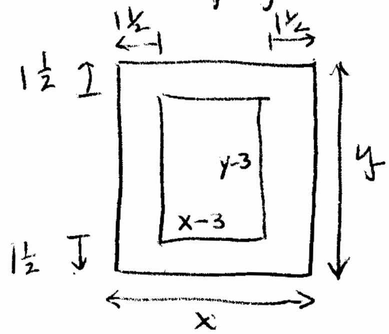
$$8\pi r^3 = 24000$$

$$r^3 = \frac{24000}{8\pi} = \frac{3000}{\pi}$$

$$r = \sqrt[3]{\frac{3000}{\pi}} = \boxed{10\sqrt[3]{\frac{3}{\pi}} = r}$$

Test: $A''(r) = \frac{8}{3}\pi + \frac{16000}{r^3}$ $\quad A''\left(\sqrt[3]{\frac{3000}{\pi}}\right) > 0$ $\cup$ MIN.

⑥ A rectangular page is to contain 36 in^2 of print. The margins on each side are $1\frac{1}{2}$". Find the dimensions of the page such that the least amount of paper is used.



amount of paper $= x \cdot y$ ← Primary function too many variables

amount of print
$(x-3)(y-3)=36$ ↙ Secondary function. Solve for y or x. I choose y

$$y-3 = \frac{36}{x-3}$$

$$y = \frac{36}{x-3} + \frac{3(x-3)}{(x-3)}$$

$$y = \frac{36+3x-9}{x-3}$$

$$y = \frac{3x+27}{x-3}$$

Subst. secondary into primary

$$P(x) = x \cdot y$$
$$= x\left(\frac{3x+27}{x-3}\right)$$

$$P(x) = \frac{3x^2+27x}{x-3}$$ ← Primary now a function of one variable

optimize.

$$P'(x) = \frac{(x-3)(6x+27)-(3x^2+27x)(1)}{(x-3)^2}$$ quotient rule

$$= \frac{6x^2+27x-18x-81-3x^2-27x}{(x-3)^2}$$

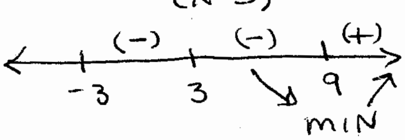
$$= \frac{3x^2-18x-81}{(x-3)^2}$$

$$= \frac{3(x^2-6x-27)}{(x-3)^2}$$

$$= \frac{3(x-9)(x+3)}{(x-3)^2}$$

$P'(x)=0$ when $x=9$, $x=-3$
$P'(x)$ undef when $x=3$
} of these, only $x=9$ is physically sensible

$P'(6)<0$
$P'(10)>0$
$P'(0)<0$



$x=9$

$$y = \frac{3(9)+27}{9-3} = 9$$

dimensions $\boxed{9" \times 9"}$

⑨ Find the speed v, in mph, that will minimize costs on a 110-mile delivery trip.

The cost per hour for fuel is $C(v) = \frac{v^2}{500}$, and the driver is paid \$7.50 per hour. Assume there are no other costs.

Need expression for time since both costs are per hour.

$D = R \cdot T$

$110 = v \cdot t$

$\frac{110}{v} = t$

$$\left.\begin{array}{l} \text{cost of fuel then is } \frac{v^2}{500} \cdot \frac{110}{v} = \frac{11v}{50} \\ \text{cost of driver then is } (7.5)\left(\frac{110}{v}\right) = \frac{825}{v} \end{array}\right\} \text{secondary functions}$$

$$\left.\begin{array}{l} \text{total cost } T(v) = \frac{11v}{50} + \frac{825}{v} \\ T(v) = \frac{11}{50}v + 825v^{-1} \end{array}\right\} \text{primary function.}$$

optimize. $T'(v) = \frac{11}{50} - 825v^{-2}$

$= \frac{11}{50} - \frac{825}{v^2}$

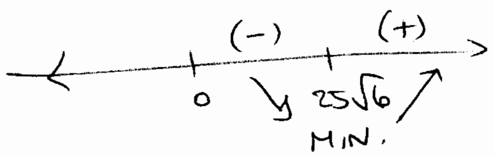
$= \frac{11v^2 - 825(50)}{50v^2}$

$= \frac{11v^2 - 41250}{50v^2}$

$= \frac{11(v^2 - 3750)}{50v^2}$

$T'(v) = 0$ when $v = \sqrt{3750} = 25\sqrt{6} \approx 61.2$ mph

$T'(v)$ undef when $v = 0$



$T'(1) < 0$

$T'(70) > 0$

a minimum in total costs occurs when $\boxed{v = 25\sqrt{6} \text{ mph}}$